A-Omega calculation on the slit micro-lens

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Abstract

AΩ change by the micro-lens to be integrated into the spectrograph entrance slit is considered. From paraxial calculations, the effective AΩ depends upon distance s between the fiber exit and the micro-lens. AΩ change is minimum with $s = r_{slit}/\theta_{core}$ when r_{slit} is radius of the imaged slit and θ_{core} is the light cone angle of the light from the fiber measured in the air. In the case, the effective AΩ becomes $(r_{core}/r_{slit} + 1)^2$ times larger.

For the smallest AΩ, focal length of the micro lens is $f = w_{slit}/(2 \arcsin NA)$ with numerical aperture of light NA (with $NA = \sin\theta_{core}$) and pseudo slit width w_{slit} (with $w_{slit} = 2r_{slit}$). If the lens is directly attached to the fiber exit, its length should be $L_{lens} = w_{slit}/(2n_{lens} \arcsin NA)$ and surface curvature radius be $c_{lens} = w_{slit} \times (n_{lens} - 1)/(2 \arcsin NA)$. Moreover, lens radius is $r_{lens} = r_{core} + f \times \arcsin NA$ and focal ratio is $F = 1/[2 \arcsin NA \times (D_{core}/w_{slit} - 1)]$.

1 Introduction

On considerations of spectrometer design[1], it was made clear that we can not afford for AΩ enlarged by the sparse core area surrounded by the cladding layers in the pseudo slit. Table 1 shows an example of the K-band spectrometer. The design assumes 50 μm diameter fiber cores and 100 μm interval to align the fibers, and the 100 μm interval is imaged on 2 pixels of the FPA. As this is a paraxial calculation, lens diameter of the camera optics is practically larger than 290 mm, which is too big to be really constructed.

A micro-lens array on the spectrometer entrance slit is considered here to minmize the AΩ required for the spectrometer.

2 Basic assumptions

A micro-lens array is assumed to be attached directly onto the fiber exits. The fiber has a core radius of r_{core} in μm, with the scrambled telescope pupil is imaged on the exit. The micro-lens images the scrambled telescope focal plane

[1] e.g. tomono@gin-an:$\tilde{}$/Design01/011229_K_Spec

Table 1: An example of K-band spectrometer design

fiber FOV	0.2 arcsec
N.A. of light	two times bigger to allow FRD
fiber core diameter	50 μm
fiber cladding diameter	100 μm
resolution power	4000
grating diffraction order	2
groove interval	12.6 μm
central wavelength	2.2 μm
width of a spectral element on FPA	37 μm
incident angle α	-4.6 deg
emergent angle β	25.4 deg
collimator focal length	1035 mm
camera focal length	382 mm
incident beam width	321 mm
grating width	322 mm
emergent beam width	290 mm

onto an image plane of radius r_{slit} in μm, which is the assumed cladding diameter. The paraxial micro-lens is assumed to be placed s (in μm) from the fiber exit, making the image plane t (in μm) from the lens. Here, effective focal length of the micro-lens f is the same as t.

3 AΩ of a ray

Figure 1 shows the assumed geometry of the fiber core and the micro-lens. Beam from the core has a light cone angle of θ_{core} in the air. Output light cone angle is calculated as follows for small θ.

As a collimated beam is focused onto a single point on the image plane, r_{slit} can be calculated as

$$r_{slit} = t\theta_{core}. \tag{1}$$

The output light cone angle θ_{slit} is calculated tracing light from upper most point and lower most point on the fiber core going in to the lens in the same direction as the optical axis:

$$\theta_{slit} = \frac{r_{core}}{t} \tag{2}$$

Therefore, AΩ at the slit for a collimated beam is

$$\begin{aligned} \mathrm{A}\Omega &= r_{slit}^2 \theta_{slit}^2 \pi^2 && (3) \\ &= r_{core}^2 \theta_{core}^2 \pi^2. && (4) \end{aligned}$$

AΩ *of a ray* on the fiber eixt is the same as on the pseudo slit.

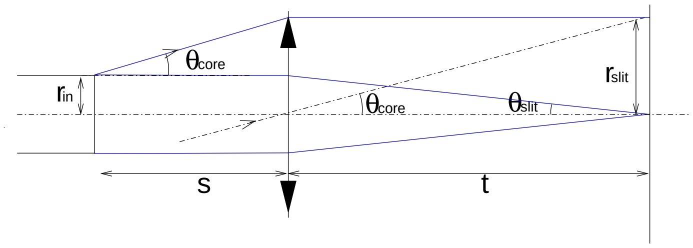

Figure 1: Geometry of the fiber exit and the micro-lens

4 Effective AΩ

We have to expand our consideration light from anywhere on the fiber core. Figure 2 shows that the beam direction θ_{center} of the light depends upon the direction of light from the fiber core.

A ray from the center with maximum divergence from the optical axis goes through the lens at

$$x = s\theta_{core} \tag{5}$$

and reaches the image plane at

$$r_{slit} = x + t\theta_{center}. \tag{6}$$

Therefore,

$$\theta_{center} = \theta_{core}\left(1 - \frac{s\theta_{core}}{r_{slit}}\right). \tag{7}$$

Beam divergence θ_{max} from the image plane is

$$\begin{aligned}
\theta_{max} &= \max\left|\theta_{center} \pm \theta_{slit}\right| && (8)\\
&= \max\left|\theta_{core}\left(1 - \frac{s\theta_{core} \mp r_{core}}{r_{slit}}\right)\right| && (9)\\
&= \begin{cases} \theta_{core} \times (r_{core}/r_{slit} + 1 - s\theta_{core}/r_{slit}) & \text{for } s < r_{slit}/\theta_{core} \\ \theta_{core} \times (r_{core}/r_{slit} - 1 + s\theta_{core}/r_{slit}) & \text{for } s > r_{slit}/\theta_{core} \end{cases} && (10)
\end{aligned}$$

5 Lens radius

When packing the fibers and micro-lens assemblies into the slit, the micro-lens radius r_{lens} determines the slit length when it is bigger than the cladding size and the image size r_{slit}. From geometry,

$$r_{lens} = r_{core} + s\theta_{core} \tag{11}$$

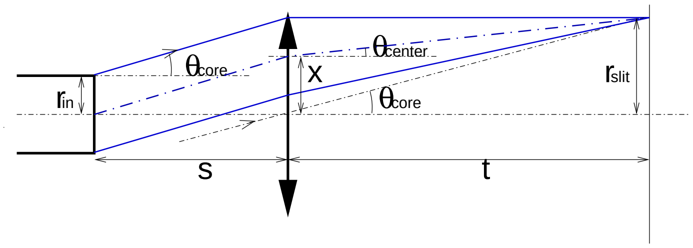

Figure 2: Geometry of the fiber exit and the micro-lens

and effective radius of the optics r_{eff} is defined as

$$r_{eff} = \max(r_{lens}, r_{slit}) \tag{12}$$

$$= \begin{cases} r_{core} \times (1 + s\theta_{core}/r_{core}) & \text{for } s > (r_{slit} - r_{core})/\theta_{core} \\ r_{slit} & \text{for } s < (r_{slit} - r_{core})/\theta_{core} \end{cases} \tag{13}$$

6 Effective AΩ

Effective AΩ of the light from the slit imaged by the micro-lens is calculated from r_{eff} and θ_{max}. Change factor k of AΩ at the fiber output (AΩ_{core}) and at the slit (AΩ_{slit}) is defined and calculated as follows:

$$k \equiv \sqrt{\frac{\mathrm{A}\Omega_{slit}}{\mathrm{A}\Omega_{core}}} \tag{14}$$

$$= \frac{r_{eff}\theta_{max}}{r_{core}\theta_{core}} \tag{15}$$

$$= \begin{cases} 1 + r_{slit}/r_{core} - s\theta_{core}/r_{core} & (s < s_1) \\ (1 + s\theta_{core}/r_{core})(r_{core}/r_{slit} + 1 - s\theta_{core}/r_{slit}) & (s_1 < s < s_2) \\ (1 + s\theta_{core}/r_{core})(r_{core}/r_{slit} - 1 + s\theta_{core}/r_{slit}) & (s_2 < s) \end{cases} \tag{16}$$

for

$$s_1 = \frac{r_{slit} - r_{core}}{\theta_{core}} \tag{17}$$

$$s_2 = \frac{r_{slit}}{\theta_{core}} \tag{18}$$

Figure 3 shows the three functions. k has two minima at $s = s_1$ and s_2 with the values of

$$k(s_1) = 2 \tag{19}$$

and

$$k(s_2) = \frac{r_{core}}{r_{slit}} + 1 \tag{20}$$

Because r_{core} is smaller than r_{slit} for our purpose, minimum value of k is at s_2. Therefore, our micro-lens should be designed so that $s = r_{slit}/\theta_{core}$.

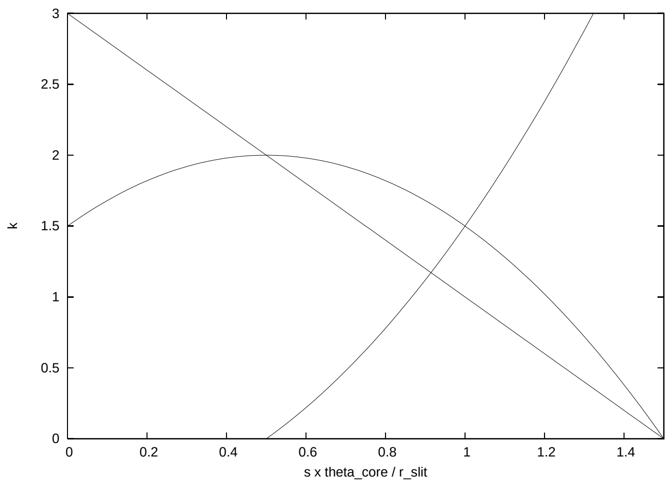

Figure 3: Effective AΩ calculated for $\theta_{max} = 0.1$, $r_{core} = 25$ μm, and $r_{slit} = 50$ μm.

7 Practical considerations about effective AΩ

Practically we will have a large number of fibers on the entrance slit. In the case, change of effective AΩ also depends upon the number of fibers we have to handle.

When N fibers with core radius r_{core} and emergent light cone angle of θ_{core} are considered, simple sum of AΩ of the light is

$$\mathrm{A}\Omega_{orig} = N \times \pi r_{core}^2 \times \pi\theta_{core}^2. \tag{21}$$

First the fibers with cladding radius r_{clad} are considered to be simply lined up on the slit. In the case, we have to extend the acceptance field of view of the spectrograph. With an on-axis system, the field of view have to be a circular with a radius of $N \times r_{clad}$. In the case effective AΩ of the light that the spectrograph has to accept is

$$\mathrm{A}\Omega_{f,c} = \pi(N r_{clad})^2 \times \pi\theta_{core}^2. \tag{22}$$

When an off-axis system is designed, the smallest field of view can be a rectangle of $N \times r_{clad}$ by r_{core}. In the case effective AΩ is

$$\mathrm{A}\Omega_{f,r} = \pi(N r_{clad} r_{core}) \times \pi\theta_{core}^2. \tag{23}$$

With a micro lens array with $s = r_{slit}/\theta_{core}$, effective A$\Omega$ of the circular field of view becomes

$$\begin{aligned} \mathrm{A}\Omega_{l,c} &= \pi(N r_{eff})^2 \times \pi\theta_{max}^2 & (24) \\ &= \pi N^2 (r_{core} + r_{slit})^2 \times \pi\theta_{core}^2 \left(\frac{r_{core}}{r_{slit}}\right)^2. & (25) \end{aligned}$$

Table 2: Change of AΩ at the spectrograph entrance slit. N of fibers with core radius of r_{core} and cladding radius of r_{clad} are assumed. For micro lens array a design with $s = r_{slit}/\theta_{core}$ is adopted with radius (half width) of the imaged slit of r_{slit}.

	circular FOV	rectangular FOV
Simple fibers	$N\left(\frac{r_{clad}}{r_{core}}\right)^2$	$\frac{4}{\pi}\frac{r_{clad}}{r_{core}}$
Micro lens	$N\left(1+\frac{r_{core}}{r_{slit}}\right)^2$	$\frac{4}{\pi}\left(1+\frac{r_{core}}{r_{slit}}\right)$

For the rectangular field case,

$$\mathrm{A}\Omega_{l,r} = \pi(2Nr_{eff} \times 2r_{slit}) \times \pi\theta_{max}^2 \tag{26}$$

$$= 4Nr_{core}\left(1+\frac{r_{slit}}{r_{core}}\right) r_{slit} \times \pi\theta_{core}^2 \left(\frac{r_{core}}{r_{slit}}\right)^2 \tag{27}$$

Dividing with $\mathrm{A}\Omega_{orig}$ changes of effective AΩ are calculated as in Table 2.

It is better to integrate a micro lens array on the spectrograph entrance slit if the pesudo slit can be wide enough:

$$\frac{r_{slit}}{r_{core}} > \frac{r_{core}}{r_{clad} - r_{core}}. \tag{28}$$

With typical values of $r_{core} = 25\mu$m and $r_{clad} = 50\mu$m, r_{slit} should be $> 25\mu$m if we are to integrate a micro lens array.

8 Lens design with the smallest effective AΩ

From Equation 20, the spectrometer get smallest effective AΩ ($r_{core}/r_{slit} + 1$ times bigger than the input light) when $s = r_{slit}/\theta_{core}$. From Equation 1, $t = r_{slit}/\theta_{core}$. Therefore, to minimize effective AΩ, $s = t$ in Figures 1 and 2.

From geometry, focal length of the micro lens f is the same as t, and defined by

$$f_{lens} = \frac{w_{slit}}{2\theta_{core}} \tag{29}$$

$$= \frac{w_{slit}}{2\arcsin NA} \tag{30}$$

where θ_{core} is the light cone angle of light in the air with numerical aperture NA we have to accept in the slit width $w_{slit} = 2 \times r_{slit}$.

We are thinking of micro lens directly attached to the fiber exit. In the case, lens length L_{lens} has to be made longer from s according to its refractive index

n_{lens}. Therefore,

$$L_{lens} = s \times n_{lens} \tag{31}$$
$$= \frac{w_{slit} \times n_{lens}}{2 \arcsin NA} \tag{32}$$

and from the geometry, the lens surface curvature radius becomes

$$c_{lens} = f(n_{lens} - 1) \tag{33}$$
$$= \frac{w_{slit} \times (n_{lens} - 1)}{2 \arcsin NA} \tag{34}$$

Moreover, from Equation 11, lens radius becomes

$$r_{lens} = r_{core} + f_{lens}\theta_{core} \tag{35}$$
$$= r_{core} + f_{lens} \times \arcsin NA \tag{36}$$

and focal ratio of the lens becomes

$$F = \frac{f_{lens}}{2 \times r_{lens}} \tag{37}$$
$$= \frac{w_{slit}}{2 \arcsin NA \times (w_{slit} + D_{core})} \tag{38}$$

with core diameter of $D_{core} = 2 \times r_{core}$.

8.1 Design example

From the equations above, an example of a slit micro-lens is designed. Parameters are shown in Table 3. Actual calculations are done in the directory `gin-an:tomono:~/Design02/0508_MicroLens/` with a ruby script shown in Table 4. A zemax simulation for the design is shown in Figure 4.

9 Revision history

2002.2.13. -

- First release

2002.5.8. -

- Error in Equation 13 corrected
- Section 8 added

2002.5.14. -

- Section 8 revised
- Section 8.1 added

2002.7.25. -

- Section 7 added

Table 3: An example of K-band spectrometer design

Input micro-lens	
Pupil image diameter D_{pupil}	$50/1.4$ μm
Fiber FOV θ_{FOV}	0.2 arcsec
Telescope diameter D_{tel}	8 m
Lens diameter D_{inlens}	300 μm
Refractive index	1.44
Lens length	1988.98 μm
Curvature radius	607.74 μm
Focal ratio	9.55
Light cone angle	0.1086 rad
Output micro-lens	
FRD parameter (output/input cone angle)	1.4
Pseudo slit width	200 μm
Fiber core diameter	50 μm
Refractive index	1.44
Lens length	947.13 μm
Curvature radius	289.40 μm
Lens diameter	250 μm
Focal ratio	2.63
Light cone angle	0.1520 rad

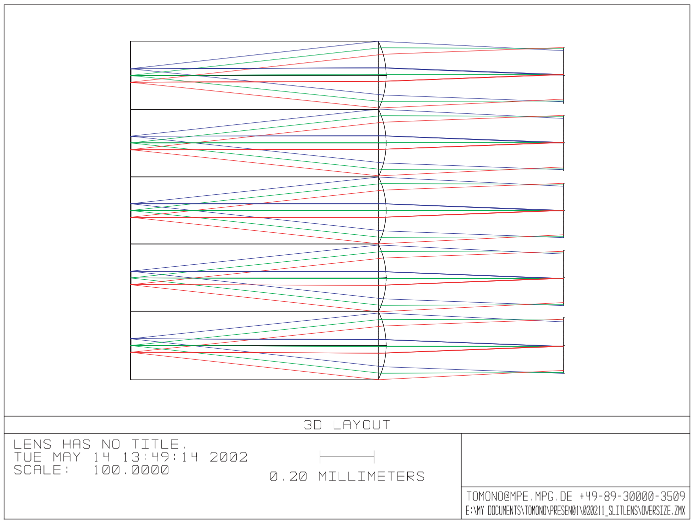

Figure 4: ZEMAX simulation of the slit micro-lens shown in Table 3

```ruby
#!/usr/bin/ruby

class InputLens
  attr_reader :d_pupil_um, :theta_fov_arcsec, :d_tel_m, :r_um, :l_um,
              :f, :theta_core_rad, :na, :n_lens
  def initialize (d_pupil_um = 50, theta_fov_arcsec = 0.2, d_tel_m = 8,
                  d_lens_um = 300, n_lens = 1.44)
    theta_fov_rad = theta_fov_arcsec * Math::PI / (180 * 3600)
    d_tel_um = d_tel_m * 1e6
    @n_lens = n_lens
    @d_pupil_um = d_pupil_um
    @theta_fov_arcsec = theta_fov_arcsec
    @d_tel_m = d_tel_m
    @r_um = d_pupil_um * (n_lens - 1) * d_lens_um / (theta_fov_rad * d_tel_um)
    @l_um = d_pupil_um * n_lens * d_lens_um / (theta_fov_rad * d_tel_um)
    @f = d_pupil_um * n_lens * n_lens / (theta_fov_rad * d_tel_um)
    @theta_core_rad = (d_tel_um * theta_fov_rad) / (2 * d_pupil_um)
    @na = Math.atan2(@theta_core_rad,
                     Math.sqrt(1-@theta_core_rad*@theta_core_rad))
  end
  def to_s
    "D_pupil:#{"%.2f" % @d_pupil_um}um L_lens:#{"%.2f" % @l_um}um\
 R_lens:#{"%.2f" % @r_um}um F:#{"%.2f" % @f} t_core:#{"%.4f" %\
 @theta_core_rad}rad"
  end
end

class OutputLens
  attr_reader :inputlens, :w_slit_um, :theta_core_rad, :d_core_um,
              :f_lens_um, :n_lens, :r_lens_um, :l_lens_um, :d_lens_um, :f
  def initialize (inputlens, frdpar = 1.4, w_slit_um = 200, d_core_um = 50,
                  n_lens = nil)
    @inputlens = inputlens
    @w_slit_um = w_slit_um
    @theta_core_rad = @inputlens.theta_core_rad * frdpar
    @f_lens_um = @w_slit_um/(2*@theta_core_rad)
    @d_core_um = d_core_um
    @n_lens = n_lens ? n_lens : @inputlens.n_lens
    @r_lens_um = @w_slit_um*(@n_lens-1)/(2*@theta_core_rad)
    @l_lens_um = @w_slit_um*@n_lens/(2*@theta_core_rad)
    @d_lens_um = @d_core_um + 2 * @f_lens_um * @theta_core_rad
    @f = @w_slit_um / (2*@theta_core_rad*(@w_slit_um+@d_core_um))
  end
  def to_s
    "w_slit:#{"%.2f" % @w_slit_um}um f:#{"%.2f" % @f_lens_um}um\
 L:#{"%.2f" % @l_lens_um}um R:#{"%.2f" % @r_lens_um}um D:#{"%.2f" %\
 @d_lens_um}um F:#{"%.2f" % @f} t_core:#{"%.4f" %\
 @theta_core_rad}rad"
  end
end

exactOutput = OutputLens.new(InputLens.new(50))
oversizeOutput = OutputLens.new(InputLens.new(50/1.4))

puts <<_END
Exact input lens
#{exactOutput.inputlens}
#{exactOutput}

Oversized input lens
#{oversizeOutput.inputlens}
#{oversizeOutput}
_END
```

Table 4: A ruby script to calculate the micro-lens geometries